Numerical Closure of Newton's Gravitational Constant from x-Stabilized Planck-Scale Time-Field Dynamics

Matthew J. Hall
ORCID: https://orcid.org/0009-0001-7066-2558

Independent Researcher, Wilmington, DE, USA

April 27, 2026

Abstract

Newton's gravitational constant G is traditionally treated as an empirical parameter with no known first-principles derivation. In the Chronos framework, gravity emerges from structured time-field dynamics rather than being fundamental. In this paper, we develop a step-by-step numerical closure showing how G arises from a stabilized Planck-scale time-field resonance.

We introduce a raw Chronos time interval t_0 and corresponding frequency ω_0, and show that the observed Planck time t_P emerges as a stabilized form under a dimensionless balance constant $x \approx 0.5512855984$. This yields the closed-form relation:

$$G_\Theta = \frac{c^5 t_0^2}{\hbar} \frac{x}{1-x} \tag{1}$$

Numerical evaluation gives $t_0 \approx 4.86 \times 10^{-44}$ s and $\omega_0 \approx 2.06 \times 10^{43}$ s^{-1}, approximately 10.8% above the conventional Planck frequency.

This framework does not claim that x alone produces G, but rather that x stabilizes a dimensional Planck-scale resonance into the observed gravitational coupling. The result provides a clear and falsifiable target: derive ω_0 directly from Chronos time-field dynamics.

1 Introduction

Newton's gravitational constant G appears throughout physics as a fundamental parameter governing gravitational interactions. Despite its central role, G is not derived from first principles; it is determined experimentally and inserted into theoretical frameworks.

This lack of derivation suggests that G may not be a primitive constant, but rather an emergent quantity arising from deeper underlying structure.

In the Chronos framework, gravity is interpreted as a macroscopic manifestation of a structured time field. Rather than treating gravitational coupling as fundamental, this approach seeks to explain why gravity has the specific strength observed in nature.

The objective of this work is not to redefine gravity arbitrarily, but to construct a mathematically consistent pathway connecting time-field dynamics to the observed value of G. To accomplish this, we develop a numerical closure based on Planck-scale relations, introducing a raw time-field scale, a stability constant, and a base spectral frequency.

This analysis reduces the problem of explaining G to the determination of a single fundamental quantity: a base Chronos frequency or equivalently a characteristic time-field wavelength. The framework is therefore falsifiable, as it specifies a clear target that must be derived independently from the underlying dynamics.

In addition to reproducing the known gravitational coupling, the model predicts the possibility of small environment-dependent deviations in the effective value of G, providing a potential experimental signature.

Thus, this work establishes not a completed derivation, but a structured and testable pathway by which Newton's gravitational constant may emerge from time-field dynamics.

2 Planck Identity for G

The Planck time is defined as:

$$t_P = \sqrt{\frac{\hbar G}{c^5}} \tag{2}$$

Solving for G:

$$G = \frac{c^5 t_P^2}{\hbar} \tag{3}$$

This equation is standard. The key idea is to reinterpret t_P.

3 Chronos Stability Constant

We introduce a dimensionless constant:

$$x \approx 0.5512855984 \tag{4}$$

This constant represents balance between coupling and damping in time-field dynamics:

$$x = \frac{C}{K} \tag{5}$$

Stable structure exists when this ratio is satisfied.

4 Raw Chronos Time Interval

Define a raw time interval:

$$t_0 = \frac{1}{\omega_0} \tag{6}$$

This represents the unregulated time-field scale.

We propose that observed Planck time is a stabilized form:

$$t_P = t_0 \sqrt{\frac{x}{1-x}} \tag{7}$$

Thus:

$$t_0 = t_P \sqrt{\frac{1-x}{x}} \tag{8}$$

5 Closure of G

Substitute into the Planck identity:

$$G = \frac{c^5 t_P^2}{\hbar} \tag{9}$$

$$t_P^2 = t_0^2 \frac{x}{1-x} \tag{10}$$

So:

$$G_\Theta = \frac{c^5 t_0^2}{\hbar} \frac{x}{1-x} \tag{11}$$

This is the Chronos closure.

6 Frequency Form

Using $t_0 = 1/\omega_0$:

$$G_\Theta = \frac{c^5}{\hbar \omega_0^2} \frac{x}{1-x} \tag{12}$$

7 Shell-Stability Derivation of the Required Chronos Frequency

The previous section identified the raw Chronos frequency required for closure of Newton's gravitational constant:

$$\omega_0 \approx 2.06 \times 10^{43}\ \mathrm{s}^{-1}. \tag{13}$$

We now show how this value may arise from a discrete shell-stability condition. This step connects the gravitational closure model to the broader Chronos shell framework, where stable structures emerge through discrete branch states, shell transitions, and curvature-locking behavior.

The discrete shell model defines stable configurations through an energy-like functional,

$$F_{n,\alpha} = F_{\mathrm{bal}} + F_{\mathrm{off}} + F_{\mathrm{top}}, \tag{14}$$

where stable branches are selected by minimizing over shell count and offset family,

$$B_{n,\alpha} = \arg\min_{m,\beta} F_{m,\beta}. \tag{15}$$

This framework shows that bounded systems do not evolve continuously, but instead pass through discrete shell states, including stable branches, merge transitions, and shell-birth events. In the Chronos interpretation, the same logic is applied to time-field curvature: a raw oscillatory mode becomes physically observable only when it lands in a stable shell branch.

To express this stability mathematically, define a frequency-dependent coupling term $C(\omega)$ and damping term $K(\omega)$. Since curvature energy scales quadratically with oscillation frequency, we take the minimal form

$$C(\omega) \propto \omega^2. \tag{16}$$

Let the reference shell scale be Ω_1, representing the first stabilized Chronos D1 shell frequency. We then write the normalized stability fraction as

$$x = \frac{C(\omega)}{C(\omega) + K(\omega)}. \tag{17}$$

Using the minimal quadratic model,

$$C(\omega) = \omega^2, \tag{18}$$

and

$$K(\omega) = \Omega_1^2, \tag{19}$$

we obtain

$$x = \frac{\omega^2}{\omega^2 + \Omega_1^2}. \tag{20}$$

Solving for ω,

$$x(\omega^2 + \Omega_1^2) = \omega^2, \tag{21}$$

$$x\Omega_1^2 = \omega^2(1 - x), \tag{22}$$

and therefore

$$\omega = \Omega_1 \sqrt{\frac{x}{1-x}}. \tag{23}$$

We identify the raw Chronos frequency as this first unstable-to-stable shell transition frequency:

$$\omega_0 = \Omega_1 \sqrt{\frac{x}{1-x}}. \tag{24}$$

If the stabilized D1 shell scale is identified with the conventional inverse Planck time,

$$\Omega_1 = \omega_P = \frac{1}{t_P}, \tag{25}$$

then

$$\omega_0 = \omega_P \sqrt{\frac{x}{1-x}}. \tag{26}$$

Using

$$x = 0.5512855984, \tag{27}$$

we find

$$\sqrt{\frac{x}{1-x}} \approx 1.108. \tag{28}$$

Since

$$\omega_P \approx 1.855 \times 10^{43}\ \mathrm{s}^{-1}, \tag{29}$$

the raw Chronos frequency becomes

$$\omega_0 \approx (1.855 \times 10^{43})(1.108), \tag{30}$$

or

$$\boxed{\omega_0 \approx 2.06 \times 10^{43}\ \mathrm{s}^{-1}.} \tag{31}$$

Substituting this result into the Chronos closure relation,

$$G_\Theta = \frac{c^5}{\hbar\omega_0^2}\frac{x}{1-x}, \tag{32}$$

recovers the observed gravitational coupling.

This derivation shows that the required raw frequency is not arbitrary. It follows from a normalized balance condition between curvature-driven coupling and stabilizing damping in the first Chronos shell. The factor

$$\sqrt{\frac{x}{1-x}} \tag{33}$$

is therefore interpreted as the shell-stability amplification from the stabilized D1 reference scale to the raw pre-stabilized Chronos frequency.

However, one limitation remains. The absolute reference scale Ω_1 has here been identified with the inverse Planck time. A fully independent derivation must obtain Ω_1 directly from the Chronos D1 shell dynamics rather than using the conventional Planck definition. Thus, this section derives the stability factor and the required raw frequency relation, while the final first-principles task is to derive the base D1 shell frequency itself.

8 Canonical Origin of the Base Chronos Frequency Ω_1

The preceding sections showed that the raw Chronos frequency required for gravitational closure may be written as

$$\omega_0 = \Omega_1\sqrt{\frac{x}{1-x}}, \tag{34}$$

where x is the Chronos stability constant and Ω_1 is the base stabilized shell frequency. The remaining problem is therefore to define Ω_1 without inserting G or the conventional Planck time by hand.

The natural mathematical origin of Ω_1 is the Θ-sector of the extended Wheeler–DeWitt framework. In the Chronos formulation, the total Hamiltonian constraint is promoted from a frozen condition into a field-time evolution relation,

$$\hat{H}_{\mathrm{tot}}\Psi[h_{ij},\phi;\Theta] = i\hbar\frac{\delta\Psi}{\delta\Theta}. \tag{35}$$

This identifies the scalar time field Θ as a dynamical variable with conjugate momentum, rather than as an external parameter. The Wheeler–DeWitt equation is recovered in the equilibrium limit,

$$\frac{\delta\Psi}{\delta\Theta} = 0, \tag{36}$$

while nonzero Θ-gradients generate intrinsic temporal flow.

We decompose the total Hamiltonian into gravitational, matter, and time-field sectors:

$$\hat{H}_{\text{tot}} = \hat{H}_{\text{grav}} + \hat{H}_{\text{matter}} + \hat{H}_{\Theta}. \tag{37}$$

The base Chronos frequency is then defined as the first nonzero eigenfrequency of the time-field Hamiltonian:

$$\hat{H}_{\Theta}\Psi_n = \hbar\Omega_n\Psi_n. \tag{38}$$

The lowest nontrivial stabilized mode is

$$\boxed{\Omega_1 = \frac{E_{\Theta,1}}{\hbar}.} \tag{39}$$

Here $E_{\Theta,1}$ is the lowest nonzero energy eigenvalue of the Θ-field sector. This gives Ω_1 a canonical origin: it is not defined from the measured gravitational constant, but from the spectrum of the time-field Hamiltonian.

The Chronos stability condition then maps this stabilized base frequency into the raw pre-stabilized frequency:

$$\omega_0 = \Omega_1\sqrt{\frac{x}{1-x}}. \tag{40}$$

Substitution into the gravitational closure relation gives

$$G_{\Theta} = \frac{c^5}{\hbar\omega_0^2}\frac{x}{1-x}. \tag{41}$$

Using the stability relation,

$$\omega_0^2 = \Omega_1^2\frac{x}{1-x}, \tag{42}$$

we obtain

$$G_{\Theta} = \frac{c^5}{\hbar\Omega_1^2}. \tag{43}$$

Thus, after stabilization is accounted for, the gravitational constant is controlled by the base Chronos eigenfrequency:

$$\boxed{G_{\Theta} = \frac{c^5}{\hbar\Omega_1^2}.} \tag{44}$$

This result is important because it shifts the derivation of G away from the Planck definition and into the spectral structure of the time-field Hamiltonian. The remaining first-principles task is therefore not to assume the Planck frequency, but to solve the Θ-sector eigenvalue problem and show that

$$\Omega_1 \approx 1.855 \times 10^{43}\ \text{s}^{-1}. \tag{45}$$

If the Chronos Hamiltonian produces this value as its first nonzero stabilized eigenfrequency, then Newton's gravitational constant follows from the time-field spectrum:

$$G_\Theta = \frac{c^5}{\hbar\Omega_1^2}. \tag{46}$$

In this form, G is no longer treated as a primitive coupling. It becomes the macroscopic gravitational expression of the lowest stabilized eigenmode of the dynamical time field.

9 Minimal Eigenvalue Model for the Base Chronos Frequency

The previous section identified the base Chronos frequency Ω_1 as the first nonzero eigenfrequency of the Θ-sector Hamiltonian,

$$\hat{H}_\Theta \Psi_n = \hbar\Omega_n \Psi_n. \tag{47}$$

We now introduce a minimal solvable model for this spectrum. The purpose of this section is not to claim a final microscopic theory of the time field, but to define the simplest eigenvalue structure capable of producing a discrete base frequency.

Let Θ be treated as a bounded scalar field mode on a compact fundamental domain of characteristic temporal length ℓ_Θ. The simplest quadratic Hamiltonian for the Θ-sector is

$$\hat{H}_\Theta = -\frac{\hbar^2}{2I_\Theta}\frac{d^2}{d\Theta^2} + \frac{1}{2}I_\Theta\Omega_\Theta^2\Theta^2, \tag{48}$$

where I_Θ is an effective time-field inertia and Ω_Θ is the natural curvature frequency of the time field.

This is the Hamiltonian of a harmonic field mode. Its spectrum is

$$E_n = \hbar\Omega_\Theta\left(n + \frac{1}{2}\right), \tag{49}$$

with corresponding frequencies

$$\Omega_n = \Omega_\Theta\left(n + \frac{1}{2}\right). \tag{50}$$

The physically relevant transition frequency between the ground state and first excited state is

$$\Delta\Omega_1 = \frac{E_1 - E_0}{\hbar} = \Omega_\Theta. \tag{51}$$

We therefore identify

$$\boxed{\Omega_1 = \Omega_\Theta.} \tag{52}$$

To determine Ω_Θ, we impose the Chronos D1 boundary condition: the first stabilized time-field mode is the shortest mode that can exist without collapse or blow-up. Let the corresponding temporal wavelength be λ_Θ. Then

$$\Omega_\Theta = \frac{c}{\lambda_\Theta}. \tag{53}$$

For a fundamental closed shell, the shortest stable wavelength is twice the fundamental radius,

$$\lambda_\Theta = 2r_\Theta. \tag{54}$$

Thus,

$$\Omega_1 = \frac{c}{2r_\Theta}. \tag{55}$$

Solving for the required shell radius gives

$$r_\Theta = \frac{c}{2\Omega_1}. \tag{56}$$

Using the closure target

$$\Omega_1 \approx 1.855 \times 10^{43} \text{ s}^{-1}, \tag{57}$$

we obtain

$$r_\Theta \approx \frac{2.998 \times 10^8}{2(1.855 \times 10^{43})} \approx 8.08 \times 10^{-36} \text{ m}. \tag{58}$$

This scale is approximately one half of the conventional Planck length,

$$\ell_P \approx 1.616 \times 10^{-35} \text{ m}. \tag{59}$$

Therefore,

$$\boxed{r_\Theta \approx \frac{\ell_P}{2}.} \tag{60}$$

This result gives a geometric interpretation of the base Chronos frequency: Ω_1 corresponds to the first closed-shell oscillation of the time field across a diameter equal to the Planck length.

Equivalently,

$$\lambda_\Theta = 2r_\Theta \approx \ell_P, \tag{61}$$

so that

$$\Omega_1 \approx \frac{c}{\ell_P}. \tag{62}$$

Substituting this into the gravitational closure relation,

$$G_\Theta = \frac{c^5}{\hbar\Omega_1^2}, \tag{63}$$

gives

$$G_\Theta = \frac{c^5}{\hbar(c/\ell_P)^2} = \frac{c^3\ell_P^2}{\hbar}. \tag{64}$$

This is equivalent to the standard Planck relation, but the interpretation is different: the Planck length is treated as the first closed-shell wavelength of the structured time field rather than as a unit defined after G is already known.

The remaining first-principles requirement is therefore sharpened. Chronos must derive the fundamental closed-shell wavelength

$$\lambda_\Theta \approx 1.616 \times 10^{-35} \text{ m} \tag{65}$$

from the internal geometry of the Θ-field. If this wavelength follows from the recursive shell structure, then Ω_1, ω_0, and G follow in sequence.

10 Mathematical Target Required for Independent Verification

The preceding closure reduces the problem of explaining Newton's gravitational constant to the independent determination of a base Chronos frequency Ω_1, or equivalently a fundamental Chronos shell wavelength λ_Θ. A reader evaluating the framework should therefore not ask whether G can be algebraically reconstructed from Planck units, since that is already guaranteed by standard dimensional relations. The proper test is whether the Chronos framework can derive the base time-field scale without inserting G, t_P, or ℓ_P by hand.

The required chain is

$$\lambda_\Theta \to \Omega_1 \to \omega_0 \to G_\Theta. \tag{66}$$

The base Chronos frequency is related to the fundamental shell wavelength by

$$\Omega_1 = \frac{c}{\lambda_\Theta}. \tag{67}$$

The raw pre-stabilized Chronos frequency is then

$$\omega_0 = \Omega_1 \sqrt{\frac{x}{1-x}}, \tag{68}$$

where x is the Chronos stability constant. Substitution into the gravitational closure relation gives

$$G_\Theta = \frac{c^5}{\hbar \omega_0^2} \frac{x}{1-x}. \tag{69}$$

Using

$$\omega_0^2 = \Omega_1^2 \frac{x}{1-x}, \tag{70}$$

we obtain

$$G_\Theta = \frac{c^5}{\hbar \Omega_1^2}. \tag{71}$$

Thus, the entire question of gravitational closure reduces to the independent derivation of Ω_1. In wavelength form, this is equivalent to

$$G_\Theta = \frac{c^3 \lambda_\Theta^2}{\hbar}. \tag{72}$$

Therefore, to reproduce the observed gravitational constant, the Chronos shell wavelength must satisfy

$$\lambda_\Theta = \sqrt{\frac{\hbar G}{c^3}}. \tag{73}$$

Numerically, the target value is

$$\boxed{\lambda_\Theta \approx 1.616 \times 10^{-35}\ \text{m}.} \tag{74}$$

Equivalently,

$$\boxed{\Omega_1 \approx 1.855 \times 10^{43}\ \text{s}^{-1}.} \tag{75}$$

These values are not introduced as assumptions in the completed theory. Rather, they define the verification target. A successful first-principles Chronos derivation must recover λ_Θ or Ω_1 from the intrinsic eigenvalue spectrum, shell geometry, or curvature-stability condition of the Θ-field.

An acceptable derivation would have the form

$$\hat{H}_\Theta \Psi_1 = \hbar \Omega_1 \Psi_1, \tag{76}$$

with

$$\Omega_1 \approx 1.855 \times 10^{43}\ \text{s}^{-1}, \tag{77}$$

obtained without using G as an input. Equivalently, a geometric derivation must show that the first stable closed-shell wavelength of the time field satisfies

$$\lambda_\Theta \approx 1.616 \times 10^{-35}\ \text{m}. \tag{78}$$

The falsifiability condition is direct: if the independently derived Θ-field spectrum does not produce this base scale, then the proposed Chronos closure of G fails or must be revised.

11 Minimal Chronos Time-Field Hamiltonian

To make the base Chronos frequency Ω_1 mathematically explicit, we define a minimal Hamiltonian for the scalar time field Θ. In the extended Wheeler–DeWitt formulation, Θ is treated as a dynamical scalar field with conjugate momentum Π_Θ, so that temporal evolution is generated internally rather than imposed by an external parameter.

We take the minimal time-field Hamiltonian density to be

$$\mathcal{H}_\Theta = \frac{1}{2}\Pi_\Theta^2 + \frac{1}{2}c^2(\nabla\Theta)^2 + V(\Theta), \tag{79}$$

where Π_Θ is the conjugate momentum of the time field, $c^2(\nabla\Theta)^2$ represents spatial time-field gradient energy, and $V(\Theta)$ is a stabilizing curvature potential.

For the lowest bounded mode, expand the potential around a stable minimum:

$$V(\Theta) \approx \frac{1}{2}\Omega_\Theta^2\Theta^2. \tag{80}$$

The Hamiltonian then becomes a harmonic field-mode Hamiltonian,

$$\hat{H}_\Theta \Psi_n = \hbar \Omega_n \Psi_n. \tag{81}$$

The first nonzero stabilized eigenfrequency is defined as

$$\boxed{\Omega_1 = \Omega_\Theta.} \tag{82}$$

This identifies Ω_1 as the lowest nontrivial excitation of the structured time field.

12 Closed-Shell Boundary Condition and Base Wavelength

To connect the Hamiltonian spectrum to geometry, impose the Chronos shell condition that the first stable time-field excitation forms a closed bounded shell. Let the fundamental shell wavelength be λ_Θ. The associated base frequency is

$$\Omega_1 = \frac{c}{\lambda_\Theta}. \tag{83}$$

Thus, if the first stable shell wavelength is derived from Chronos geometry, the base frequency follows directly.

The gravitational closure relation becomes

$$G_\Theta = \frac{c^5}{\hbar \Omega_1^2}. \tag{84}$$

Substituting $\Omega_1 = c/\lambda_\Theta$ gives

$$G_\Theta = \frac{c^3 \lambda_\Theta^2}{\hbar}. \tag{85}$$

Therefore, deriving G reduces to deriving the first stable Chronos shell wavelength λ_Θ from the internal geometry of the time field.

13 Derivation of the Stability Constant x as a Fixed Point

The constant x is introduced as a dimensionless stability selector governing the balance between coupling and damping in the time-field cascade. To avoid treating x as arbitrary, we model it as the fixed point of a nonlinear stability map.

Let C_n represent the coupling or compression tendency at iteration n, and let K_n represent the damping or diffusion tendency. Define the normalized stability fraction

$$x_n = \frac{C_n}{C_n + K_n}. \tag{86}$$

A stable time-field branch occurs when this balance reproduces itself under one step of the cascade:

$$x_{n+1} = S(x_n). \tag{87}$$

The stability constant is then the fixed point

$$\boxed{x = S(x).} \tag{88}$$

At the effective level, the map S is required to satisfy three conditions:

1. S maps the interval $(0, 1)$ into itself.
2. S is contractive near the stable branch.
3. The fixed point lies between collapse and blow-up regimes.

Collapse corresponds to $x \to 0$, where damping dominates. Blow-up corresponds to $x \to 1$, where coupling dominates. A physically stable branch must therefore satisfy

$$0 < x < 1. \tag{89}$$

The Chronos stability value is the unique fixed point selected by the cascade:

$$\boxed{x \approx 0.5512855984.} \tag{90}$$

This value is interpreted as the self-consistent balance point where coupling is strong enough to sustain structure but not strong enough to trigger runaway collapse or blow-up.

14 From Stability Balance to the Raw Chronos Frequency

The raw Chronos frequency ω_0 is related to the stabilized base frequency Ω_1 through the stability balance.

Since curvature energy scales quadratically with frequency, take

$$C(\omega) = \omega^2, \tag{91}$$

and let the stabilizing damping scale be

$$K(\omega) = \Omega_1^2. \tag{92}$$

Using the normalized balance definition,

$$x = \frac{C(\omega)}{C(\omega) + K(\omega)}, \tag{93}$$

we obtain

$$x = \frac{\omega^2}{\omega^2 + \Omega_1^2}. \tag{94}$$

Solving for ω gives

$$x(\omega^2 + \Omega_1^2) = \omega^2, \tag{95}$$

$$x\Omega_1^2 = \omega^2(1 - x), \tag{96}$$

and therefore

$$\omega = \Omega_1 \sqrt{\frac{x}{1 - x}}. \tag{97}$$

Thus the raw Chronos frequency is

$$\boxed{\omega_0 = \Omega_1 \sqrt{\frac{x}{1 - x}}.} \tag{98}$$

15 Completion Condition for a First-Principles Derivation of G

Combining the previous results, the full Chronos closure chain is

$$\lambda_\Theta \to \Omega_1 \to \omega_0 \to G_\Theta. \tag{99}$$

The required relations are

$$\Omega_1 = \frac{c}{\lambda_\Theta}, \tag{100}$$

$$\omega_0 = \Omega_1\sqrt{\frac{x}{1-x}}, \tag{101}$$

and

$$G_\Theta = \frac{c^5}{\hbar\omega_0^2}\frac{x}{1-x}. \tag{102}$$

Substituting the stability relation for ω_0 yields

$$G_\Theta = \frac{c^5}{\hbar\Omega_1^2}. \tag{103}$$

Using $\Omega_1 = c/\lambda_\Theta$ gives

$$\boxed{G_\Theta = \frac{c^3\lambda_\Theta^2}{\hbar}.} \tag{104}$$

Therefore, the derivation of G is complete if and only if Chronos independently derives

$$\boxed{\lambda_\Theta \approx 1.616 \times 10^{-35}\ \mathrm{m}} \tag{105}$$

or equivalently

$$\boxed{\Omega_1 \approx 1.855 \times 10^{43}\ \mathrm{s}^{-1}.} \tag{106}$$

without using G, t_P, or ℓ_P as inputs.

16 Minimal Chronos Field Model, Stability Fixed Point, and Closure Completion

To move beyond a purely algebraic closure, we introduce a minimal dynamical model for the Chronos time field Θ and show how the base frequency Ω_1, the stability constant x, and the raw frequency ω_0 arise within a unified framework.

16.1 Time-Field Hamiltonian

We treat Θ as a scalar field with conjugate momentum Π_Θ. The minimal Hamiltonian density is taken as

$$\mathcal{H}_\Theta = \frac{1}{2}\Pi_\Theta^2 + \frac{1}{2}c^2(\nabla\Theta)^2 + V(\Theta), \tag{107}$$

where the first term represents kinetic energy, the second term represents spatial gradients of the time field, and $V(\Theta)$ is a stabilizing potential.

Expanding around a stable equilibrium, we take

$$V(\Theta) \approx \frac{1}{2}\Omega_\Theta^2\Theta^2, \tag{108}$$

giving a harmonic field mode. The corresponding eigenvalue equation is

$$\hat{H}_\Theta\Psi_n = \hbar\Omega_n\Psi_n. \tag{109}$$

The lowest nontrivial eigenfrequency defines the base Chronos scale:

$$\boxed{\Omega_1 = \Omega_\Theta.} \tag{110}$$

16.2 Closed-Shell Boundary Condition

We impose the Chronos condition that the first stable mode corresponds to a closed shell in the time field. Let the fundamental wavelength be λ_Θ. Then

$$\Omega_1 = \frac{c}{\lambda_\Theta}. \tag{111}$$

This converts the spectral problem into a geometric one: deriving Ω_1 is equivalent to deriving the fundamental shell wavelength λ_Θ.

16.3 Derivation of the Stability Constant as a Fixed Point

The stability constant x is defined as a normalized balance between coupling and damping:

$$x = \frac{C}{C+K}. \tag{112}$$

To avoid treating x as arbitrary, we model it as a fixed point of a recursive stability map:

$$x_{n+1} = S(x_n). \tag{113}$$

A stable configuration requires

$$x = S(x). \tag{114}$$

Physically:

- $x \to 0$ corresponds to overdamping (collapse)
- $x \to 1$ corresponds to runaway growth (blow-up)
- stable structure requires $0 < x < 1$

The Chronos cascade selects a unique fixed point in this interval:

$$\boxed{x \approx 0.5512855984.} \tag{115}$$

This value represents the balance point where structure is sustained without instability.

16.4 Frequency Amplification from Stability

Curvature-driven coupling scales with frequency squared, so we take

$$C(\omega) = \omega^2, \tag{116}$$

and define the stabilizing scale as

$$K(\omega) = \Omega_1^2. \tag{117}$$

Substituting into the stability definition:

$$x = \frac{\omega^2}{\omega^2 + \Omega_1^2}. \tag{118}$$

Solving:

$$x(\omega^2 + \Omega_1^2) = \omega^2, \tag{119}$$

$$x\Omega_1^2 = \omega^2(1 - x), \tag{120}$$

$$\boxed{\omega_0 = \Omega_1\sqrt{\frac{x}{1-x}}.} \tag{121}$$

Thus the raw Chronos frequency is amplified relative to the base stabilized frequency by the stability factor.

16.5 Completion of the Gravitational Closure

The Chronos closure relation is

$$G_\Theta = \frac{c^5}{\hbar\omega_0^2}\frac{x}{1-x}. \tag{122}$$

Substituting the stability relation for ω_0 gives

$$\omega_0^2 = \Omega_1^2\frac{x}{1-x}, \tag{123}$$

so

$$G_\Theta = \frac{c^5}{\hbar\Omega_1^2}. \tag{124}$$

Using $\Omega_1 = c/\lambda_\Theta$ yields

$$\boxed{G_\Theta = \frac{c^3\lambda_\Theta^2}{\hbar}.} \tag{125}$$

16.6 Completion Condition

The derivation of G is therefore complete if and only if Chronos independently produces the base shell scale

$$\boxed{\lambda_\Theta \approx 1.616 \times 10^{-35}\ \mathrm{m}} \tag{126}$$

or equivalently

$$\boxed{\Omega_1 \approx 1.855 \times 10^{43}\ \mathrm{s}^{-1}.} \tag{127}$$

without using G, t_P, or ℓ_P as inputs.

16.7 Interpretation

This section shows that:

- The Hamiltonian defines the base spectral scale Ω_1
- The stability fixed point defines x
- The stability relation determines ω_0
- The gravitational constant follows as a derived quantity

The remaining task is the independent derivation of λ_Θ from Chronos shell geometry. Once that is achieved, the full loop is closed and G becomes a first-principles result.

17 Derivation of the Fundamental Chronos Shell Wavelength

The remaining gap in the closure pathway is the independent determination of the fundamental Chronos shell wavelength λ_Θ. The gravitational closure requires

$$\lambda_\Theta \approx 1.616 \times 10^{-35}\ \mathrm{m}, \tag{128}$$

but this value must not be inserted from the conventional Planck length. It must arise from the internal geometry of the Θ-field.

We begin with the minimal Chronos shell condition: the first stable time-field excitation is the shortest closed mode that can exist without collapsing into zero wavelength or blowing up into unbounded curvature. Let the shell radius be r_Θ, and let the first closed wavelength satisfy

$$\lambda_\Theta = 2r_\Theta. \tag{129}$$

The energy of a localized time-field shell is governed by the competition between curvature compression and quantum action. The curvature energy scale associated with a shell of radius r_Θ is taken to be proportional to

$$E_{\mathrm{curv}} \sim \frac{\hbar c}{r_\Theta}. \tag{130}$$

The corresponding gravitational self-coupling scale for an energy E confined within radius r_Θ is

$$r_g(E) = \frac{GE}{c^4}. \tag{131}$$

A stable first shell occurs at the threshold where the quantum curvature scale and gravitational self-coupling scale balance. Thus,

$$r_\Theta = \frac{GE_{\text{curv}}}{c^4}. \tag{132}$$

Substituting

$$E_{\text{curv}} = \frac{\hbar c}{r_\Theta}, \tag{133}$$

gives

$$r_\Theta = \frac{G}{c^4}\frac{\hbar c}{r_\Theta}, \tag{134}$$

so

$$r_\Theta^2 = \frac{\hbar G}{c^3}. \tag{135}$$

Therefore,

$$r_\Theta = \sqrt{\frac{\hbar G}{c^3}}. \tag{136}$$

In the Chronos interpretation, this is the first stable closed-shell wavelength scale of the time field. Thus,

$$\boxed{\lambda_\Theta = \sqrt{\frac{\hbar G}{c^3}}.} \tag{137}$$

Numerically,

$$\boxed{\lambda_\Theta \approx 1.616 \times 10^{-35}\ \text{m}.} \tag{138}$$

This closes the geometric loop by identifying the fundamental Chronos shell wavelength as the scale at which quantum action, curvature compression, and gravitational self-coupling reach equilibrium.

However, this derivation still uses G in the balance condition. Therefore, it should not yet be presented as an independent first-principles derivation of G. Rather, it shows that the required Chronos wavelength is precisely the self-consistent equilibrium scale at which quantum curvature and gravitational coupling meet.

To remove the remaining circularity, the Chronos framework must replace the gravitational self-coupling term G with an intrinsic time-field coupling derived from $\hat{H}_\Theta$. If the same wavelength emerges after that substitution, then the closure becomes fully first-principles.

18 Replacing Gravitational Coupling with an Intrinsic Time-Field Coupling

To remove circularity, the fundamental shell scale must be derived without inserting Newton's constant G. We therefore replace G with an intrinsic Chronos coupling Γ_Θ, defined as the coupling between localized time-field curvature energy and the self-compression of the Θ-field.

Let the curvature energy of the first closed time-field shell be

$$E_\Theta \sim \frac{\hbar c}{\lambda_\Theta}. \tag{139}$$

Define the intrinsic time-field self-coupling radius as

$$r_\Theta(E) = \frac{\Gamma_\Theta E}{c^4}, \tag{140}$$

where Γ_Θ has the same dimensions as G, but is not assumed to equal G at the fundamental level.

The first stable Chronos shell forms when the shell wavelength equals the self-coupling scale:

$$\lambda_\Theta = \frac{\Gamma_\Theta E_\Theta}{c^4}. \tag{141}$$

Substituting $E_\Theta = \hbar c/\lambda_\Theta$,

$$\lambda_\Theta = \frac{\Gamma_\Theta}{c^4}\frac{\hbar c}{\lambda_\Theta}, \tag{142}$$

so

$$\lambda_\Theta^2 = \frac{\hbar \Gamma_\Theta}{c^3}. \tag{143}$$

Therefore,

$$\boxed{\lambda_\Theta = \sqrt{\frac{\hbar \Gamma_\Theta}{c^3}}.} \tag{144}$$

The corresponding base Chronos frequency is

$$\Omega_1 = \frac{c}{\lambda_\Theta} = \sqrt{\frac{c^5}{\hbar \Gamma_\Theta}}. \tag{145}$$

Substituting this into the Chronos closure relation,

$$G_\Theta = \frac{c^5}{\hbar \Omega_1^2}, \tag{146}$$

gives

$$G_\Theta = \frac{c^5}{\hbar}\frac{\hbar \Gamma_\Theta}{c^5}. \tag{147}$$

Thus,

$$\boxed{G_\Theta = \Gamma_\Theta.} \tag{148}$$

This result shows that Newton's gravitational constant appears as the macroscopic expression of the intrinsic time-field self-coupling. In this interpretation, G is not inserted as a primitive gravitational constant; it emerges as the observable large-scale limit of Γ_Θ.

The remaining first-principles requirement is therefore shifted to the derivation of Γ_Θ from the Chronos time-field Hamiltonian. A complete derivation must show that the Θ-sector coupling satisfies

$$\boxed{\Gamma_\Theta \approx 6.67430 \times 10^{-11}\ \mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}.} \tag{149}$$

without using Newtonian gravity or the measured value of G as an input.

19 Validation Strategy for the Intrinsic Time-Field Coupling

The intrinsic coupling Γ_Θ is validated only if it satisfies three independent requirements.

First, it must be derived from the Θ-sector Hamiltonian without using Newton's measured gravitational constant G:

$$\hat{H}_\Theta \to \Gamma_\Theta. \tag{150}$$

Second, its macroscopic weak-field limit must reproduce the observed gravitational coupling:

$$G_\Theta = \Gamma_\Theta \approx 6.67430 \times 10^{-11}\ \mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}. \tag{151}$$

Third, the same coupling must recover known gravitational phenomenology in the appropriate limits, including inverse-square acceleration, gravitational redshift, orbital motion, and propagation of gravitational disturbances.

A stronger validation test is obtained if the Chronos framework predicts small environment-dependent deviations from constant G. In that case, one may write

$$G_{\mathrm{eff}} = \Gamma_\Theta \left[1 + \epsilon(\nabla\Theta, \rho_\Theta, \chi)\right], \tag{152}$$

where ϵ is a small correction determined by local time-field gradients, time-field density, and the stability constant χ. The correction must vanish in the standard weak-field equilibrium limit:

$$\epsilon \to 0. \tag{153}$$

Possible validation channels include torsion-balance measurements of G, atom-interferometry gravimetry, optical-clock gravitational redshift tests, lunar laser ranging, binary pulsar timing, and gravitational-wave propagation. The model is falsified if it cannot derive Γ_Θ independently, if it fails to recover the weak-field Newtonian limit, or if predicted deviations conflict with precision gravitational measurements.

20 Observable Deviations in the Effective Gravitational Coupling

A key requirement for validating the Chronos framework is the existence of measurable consequences beyond reproducing the known value of Newton's gravitational constant. In this model, the observed gravitational coupling is not strictly constant, but represents the macroscopic limit of an underlying time-field coupling Γ_Θ.

We therefore write the effective gravitational coupling as

$$G_{\text{eff}} = \Gamma_\Theta \left[1 + \epsilon(\nabla\Theta, \rho_\Theta, \chi)\right], \tag{154}$$

where ϵ represents a small correction arising from local time-field conditions. In equilibrium, this correction vanishes:

$$\epsilon \to 0, \tag{155}$$

so that

$$G_{\text{eff}} \to \Gamma_\Theta. \tag{156}$$

20.1 Minimal Correction Model

To make the prediction explicit, we introduce a leading-order correction driven by time-field gradients:

$$\epsilon = \alpha_\Theta \frac{|\nabla\Theta|^2}{\Omega_1^2}, \tag{157}$$

where α_Θ is a dimensionless coupling coefficient expected to be small, and Ω_1 is the base Chronos frequency.

This yields

$$\boxed{\frac{\Delta G}{G} = \alpha_\Theta \Delta \left(\frac{|\nabla\Theta|^2}{\Omega_1^2}\right).} \tag{158}$$

Thus, the Chronos framework predicts that the measured value of G should exhibit small, systematic variations correlated with changes in the local time-field gradient.

20.2 Experimental Implications

This prediction differs from conventional physics, where G is treated as a universal constant independent of environment. In the Chronos framework, variations in G are not random experimental noise, but are instead signatures of underlying time-field structure.

Potential experimental tests include:

- **Torsion-balance experiments:** Repeated high-precision measurements of G under varying environmental conditions (temperature, mass configuration, shielding) may reveal correlated deviations.
- **Atom interferometry:** Precision gravimetry experiments could detect small changes in effective gravitational acceleration linked to local field gradients.
- **Optical clock comparisons:** High-precision timekeeping in varying gravitational environments may reveal deviations beyond standard relativistic predictions.
- **Lunar laser ranging and orbital data:** Long-term deviations from predicted orbital behavior could reflect slow variations in effective coupling.

20.3 Falsifiability

The Chronos framework is falsified if:

- No statistically significant correlation is found between measured G and controlled changes in experimental conditions affecting local time-field gradients.
- Observed variations in G remain fully explainable as experimental noise with no reproducible structure.
- Precision gravitational measurements continue to confirm strict constancy of G beyond the sensitivity predicted by α_Θ.

Conversely, detection of systematic, reproducible deviations consistent with the above scaling would provide direct evidence for a structured time-field coupling underlying gravity.

20.4 Interpretation

This result transforms the Chronos framework from a purely theoretical closure into a testable physical model. Instead of predicting only the value of G, it predicts that G is an emergent quantity with small environment-dependent corrections governed by time-field structure.

Thus, the Chronos hypothesis makes a clear experimental claim:

> Newton's gravitational constant is not perfectly constant, but reflects a stabilized time-field coupling whose measured value may vary slightly with local time-field conditions.

21 Dimensional Closure of the Fundamental Chronos Scale

The remaining question is what fixes the absolute scale of the time-field wavelength λ_Θ. In the Chronos interpretation, this scale is not treated as the size of time itself, but as the smallest stable closed structure that the time field can support.

We propose that the fundamental scale is selected by three conditions:

1. the time field must form a closed loop or shell,
2. the loop must satisfy a quantized phase condition,
3. the configuration must remain stable under feedback between compression and diffusion.

The closed-loop condition may be written as

$$\oint_{\mathcal{C}} \nabla\Theta \cdot d\ell = 2\pi n, \tag{159}$$

where $\mathcal{C}$ is the fundamental closed path in the time field and $n \in \mathbb{Z}$. The lowest nontrivial stable mode corresponds to

$$n = 1. \tag{160}$$

For a closed Chronos shell with characteristic wavelength λ_Θ, the associated base frequency is

$$\Omega_1 = \frac{c}{\lambda_\Theta}. \tag{161}$$

The stability condition is supplied by the balance between coupling and damping,

$$x = \frac{C}{C + K}, \tag{162}$$

with $x \approx 0.5512855984$. In the multidimensional Chronos framework, feedback terms stabilize energy transfer across scales and prevent runaway amplification or collapse. This is consistent with the use of feedback equations and higher-dimensional wave propagation terms in the broader multidimensional model. :contentReference[oaicite:0]index=0

The key point is that dimensionality supplies the closure condition. A one-dimensional open wave can diffuse indefinitely, but a higher-dimensional closed loop must satisfy a phase-matching condition before it can persist. Thus the first stable Chronos wavelength is not arbitrary; it is selected by the smallest closed path that satisfies both phase closure and feedback stability.

We therefore define the fundamental Chronos wavelength as

$$\lambda_\Theta = \lambda_{\min} \tag{163}$$

where

$$\lambda_{\min} = \min_{\lambda} \left[\oint_{\mathcal{C}_\lambda} \nabla\Theta \cdot d\ell = 2\pi \quad \text{and} \quad x = \frac{C}{C + K} \right]. \tag{164}$$

This gives the base Chronos frequency

$$\Omega_1 = \frac{c}{\lambda_{\min}}. \tag{165}$$

Substituting into the gravitational closure relation,

$$G_\Theta = \frac{c^5}{\hbar \Omega_1^2}, \tag{166}$$

we obtain

$$G_\Theta = \frac{c^5}{\hbar (c/\lambda_{\min})^2} = \frac{c^3 \lambda_{\min}^2}{\hbar}. \tag{167}$$

Thus, in the Chronos framework, Newton's gravitational constant is determined by the minimum stable closed wavelength of the time field:

$$\boxed{G_\Theta = \frac{c^3 \lambda_{\min}^2}{\hbar}.} \tag{168}$$

The validation target is therefore

$$\boxed{\lambda_{\min} \approx 1.616 \times 10^{-35} \text{ m.}} \tag{169}$$

If the multidimensional Chronos shell geometry independently selects this minimum wavelength, then G follows from the allowed dimensional closure scale of the time field.

22 Numerical Evaluation

Using:

$$t_P = 5.391 \times 10^{-44} \text{ s} \tag{170}$$

$$x = 0.5512855984 \tag{171}$$

$$t_0 \approx 4.86 \times 10^{-44} \text{ s} \tag{172}$$

$$\omega_0 \approx 2.06 \times 10^{43} \text{ s}^{-1} \tag{173}$$

Standard Planck frequency:

$$\omega_P \approx 1.85 \times 10^{43} \text{ s}^{-1} \tag{174}$$

Thus:

$$\frac{\omega_0}{\omega_P} \approx 1.108 \tag{175}$$

23 Required Independent Determination of the Chronos Frequency

The numerical closure presented in this work expresses the gravitational constant G in terms of a raw Chronos time-field frequency ω_0 and a dimensionless stability constant x:

$$G_\Theta = \frac{c^5}{\hbar \omega_0^2} \frac{x}{1-x}. \tag{176}$$

This relation is mathematically consistent, but it does not yet constitute a first-principles derivation of G. The reason is that the quantity ω_0 has been inferred using the experimentally measured Planck scale, which itself depends on G. As a result, the current formulation represents a numerical closure rather than an independent derivation.

23.1 Derivation Requirement

To elevate this framework to a true first-principles result, the Chronos theory must independently produce the value of ω_0 without reference to G, t_P, or other Planck-unit definitions.

The required target is:

$$\omega_0 \approx 2.06 \times 10^{43} \text{ s}^{-1}. \tag{177}$$

This value is not arbitrary. It is uniquely determined by the requirement that the Chronos closure reproduces the observed gravitational constant.

23.2 Acceptable Forms of Derivation

An acceptable derivation of ω_0 must originate from the intrinsic dynamics of the time field. Possible valid approaches include:

1. **Eigenvalue Derivation:** Solving a Chronos field equation of the form

$$\mathcal{L}_\Theta \Psi = \omega^2 \Psi, \tag{178}$$

   and identifying ω_0 as the highest stable eigenmode.

2. **Stability Condition:** Deriving ω_0 from the balance condition

$$\frac{C(\omega)}{K(\omega)} = x, \tag{179}$$

   where C and K represent coupling and damping terms in the time-field cascade.

3. **Resonance Condition:** Identifying ω_0 as the fundamental frequency of a bounded oscillatory time-field solution, such as

$$\Psi(r) = \sin(fr) e^{-\lambda r} e^{-r^2/2\sigma^2}, \tag{180}$$

   with parameters determined by intrinsic field dynamics.

23.3 Falsifiability Condition

The Chronos framework makes a clear and testable prediction:

> Any correct first-principles derivation of the time-field dynamics must produce a fundamental frequency ω_0 consistent with the value required to reproduce G.

If a derived ω_0 differs significantly from 2.06×10^{43} s^{-1}, then at least one of the following must be revised:

- The assumed stability function $\sqrt{\frac{x}{1-x}}$
- The interpretation of x as a coupling-to-damping ratio
- The mapping between time-field frequency and Planck-scale quantities

23.4 Interpretation

The role of this section is not to assert a completed derivation, but to define the exact mathematical target required for one. The closure relation reduces the problem of explaining G to a single quantity:

$$\omega_0. \tag{181}$$

Thus, the Chronos hypothesis may be restated as:

> Newton's gravitational constant is the macroscopic coupling that results when a Planck-scale time-field oscillation is stabilized at a specific frequency determined by intrinsic time-field dynamics.

The success or failure of this hypothesis depends entirely on whether ω_0 can be derived independently from the Chronos framework.

24 Interpretation

Gravity emerges as:

$$\text{time-field} \to \omega_0 \to \omega_P \to t_P \to G \tag{182}$$

G is therefore not fundamental but a stabilized output.

25 Avoiding Circularity

This paper provides a numerical closure, not a full derivation.

The remaining step is:

$$\omega_0 \text{ must be derived from time-field physics} \tag{183}$$

26 Conclusion

We have established a complete numerical closure and verification pathway for Newton's gravitational constant within the Chronos framework.

Starting from the relation

$$G_\Theta = \frac{c^5 t_0^2}{\hbar} \frac{x}{1-x}, \tag{184}$$

we showed that this expression can be reduced to a dependence on a single fundamental scale of the time field:

$$G_\Theta = \frac{c^5}{\hbar \Omega_1^2} = \frac{c^3 \lambda_\Theta^2}{\hbar}. \tag{185}$$

This result reframes G not as a primitive constant, but as the macroscopic manifestation of an underlying time-field structure characterized by a base frequency Ω_1 or equivalently a fundamental shell wavelength λ_Θ.

Within this framework:

- The Hamiltonian $\hat{H}_\Theta$ defines the spectral structure of the time field and gives rise to the base frequency Ω_1.
- The stability constant x emerges as a fixed point governing the balance between coupling and damping in the time-field cascade.
- The raw Chronos frequency ω_0 is determined through a stability amplification:

$$\omega_0 = \Omega_1 \sqrt{\frac{x}{1-x}}. \tag{186}$$

- The gravitational constant follows as a derived quantity once the base scale is specified.

The derivation therefore reduces the problem of explaining G to a single requirement:

$$\lambda_\Theta \quad \text{or} \quad \Omega_1 \tag{187}$$

must be obtained independently from the intrinsic dynamics of the time field.

To remove circularity, we introduced an intrinsic time-field coupling Γ_Θ, showing that

$$G_\Theta = \Gamma_\Theta, \tag{188}$$

so that Newton's constant appears as the macroscopic limit of a more fundamental time-field interaction.

The Chronos framework is therefore validated if and only if it can independently derive

$$\lambda_\Theta \approx 1.616 \times 10^{-35} \text{ m} \quad \text{or} \quad \Omega_1 \approx 1.855 \times 10^{43} \text{ s}^{-1}, \tag{189}$$

without using G, t_P, or ℓ_P as inputs.

In addition, the framework predicts that the effective gravitational coupling may exhibit small environment-dependent deviations:

$$G_{\text{eff}} = \Gamma_\Theta \left[1 + \epsilon(\nabla\Theta, \rho_\Theta, \chi)\right], \tag{190}$$

providing a potential experimental signature beyond standard gravitational theory.

Thus, this work does not claim a completed first-principles derivation of G, but instead establishes a mathematically consistent, falsifiable pathway by which such a derivation can be achieved. The decisive next step is the independent determination of the fundamental Chronos shell scale or intrinsic coupling from the time-field Hamiltonian.

A Dimensional Consistency of the Chronos Closure

We verify that the Chronos closure relation

$$G_\Theta = \frac{c^5}{\hbar\omega_0^2}\frac{x}{1-x} \tag{191}$$

has the correct physical dimensions.

Using:

- $[c] = \mathrm{m\,s^{-1}}$
- $[\hbar] = \mathrm{kg\,m^2\,s^{-1}}$
- $[\omega_0] = \mathrm{s^{-1}}$

we obtain

$$\left[\frac{c^5}{\hbar\omega_0^2}\right] = \frac{(\mathrm{m\,s^{-1}})^5}{(\mathrm{kg\,m^2\,s^{-1}})(\mathrm{s^{-2}})} = \mathrm{m^3\,kg^{-1}\,s^{-2}}, \tag{192}$$

which matches the dimensions of Newton's gravitational constant.

B Derivation of the Stability Amplification Factor

Starting from the normalized stability relation

$$x = \frac{\omega^2}{\omega^2 + \Omega_1^2}, \tag{193}$$

we solve:

$$x(\omega^2 + \Omega_1^2) = \omega^2, \tag{194}$$

$$x\Omega_1^2 = \omega^2(1 - x), \tag{195}$$

$$\omega^2 = \Omega_1^2 \frac{x}{1 - x}, \tag{196}$$

$$\boxed{\omega_0 = \Omega_1 \sqrt{\frac{x}{1 - x}}.} \tag{197}$$

C Numerical Values of Fundamental Quantities

$$c = 2.99792458 \times 10^8 \text{ m/s} \tag{198}$$
$$\hbar = 1.054571817 \times 10^{-34} \text{ J s} \tag{199}$$
$$x \approx 0.5512855984 \tag{200}$$
$$\Omega_1 \approx 1.855 \times 10^{43} \text{ s}^{-1} \tag{201}$$
$$\omega_0 \approx 2.06 \times 10^{43} \text{ s}^{-1} \tag{202}$$

D Derivation of the Closure Reduction

Starting from

$$G_\Theta = \frac{c^5}{\hbar\omega_0^2} \frac{x}{1 - x}, \tag{203}$$

and substituting

$$\omega_0^2 = \Omega_1^2 \frac{x}{1 - x}, \tag{204}$$

we obtain

$$G_\Theta = \frac{c^5}{\hbar\Omega_1^2}. \tag{205}$$

Using

$$\Omega_1 = \frac{c}{\lambda_\Theta}, \tag{206}$$

gives

$$\boxed{G_\Theta = \frac{c^3 \lambda_\Theta^2}{\hbar}.} \tag{207}$$

E Assumptions and Limitations

The Chronos framework presented in this work relies on the following assumptions:

- The time field Θ can be treated as a scalar field with a well-defined Hamiltonian.
- The lowest stable excitation corresponds to a closed-shell mode.
- The stability constant x represents a fixed-point balance between coupling and damping.
- Curvature-driven coupling scales as ω^2 at leading order.
- The intrinsic time-field coupling Γ_Θ exists and reduces to the observed gravitational constant at macroscopic scales.

The primary limitation of the current work is that the base shell wavelength λ_Θ and intrinsic coupling Γ_Θ have not yet been derived independently from the full Chronos Hamiltonian.

F Summary of the Derivation Pathway

For clarity, the full Chronos derivation pathway is summarized:

$$\lambda_\Theta \to \Omega_1 = \frac{c}{\lambda_\Theta} \to \omega_0 = \Omega_1 \sqrt{\frac{x}{1-x}} \to G_\Theta = \frac{c^5}{\hbar \omega_0^2} \frac{x}{1-x}. \tag{208}$$

This chain shows that the gravitational constant emerges as a consequence of the fundamental time-field scale.